-: COMPOUND PENDULUM :-

let the compound pendulum be suspended from S with C as center of mass. It is oscillating in the vertical plane which is the plane of the paper.

Moment of inertia of the pendulum about the axis of rotation through S is

$$I = I_c + Ml^2 = M(K^2 + l^2)$$

Where M is the mass of the pendulum, $I_c = MK^2$ (K = radius of gyration) about a parallel axis through C and l the distance between centre of suspension and centre of mass. If θ is the instantaneous angle which 'SC' makes with the vertical axis through 'O' then the kinetic energy of the oscillating system is

$$T = \frac{1}{2} I\dot{\theta}^2 = \frac{1}{2}M(K^2+l^2)\dot{\theta}^2 \quad \text{——— (1)}$$

potential energy with respect to horizontal plane through S is

$$V = -Mgl\cos\theta \quad \text{——— (2)}$$

Langrangian $L = T - V$

$$L = \frac{1}{2}M(K^2+l^2)\dot{\theta}^2 + Mgl\cos\theta$$

or, $$L = \frac{1}{2}M(K^2+l^2)\dot{\theta}^2 + Mgl\cos\theta$$

Now, $$\frac{\partial L}{\partial \theta} = -Mgl\sin\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = M(K^2+l^2)\dot{\theta}$$

from Langrange's equation for conservative system,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0.$$

$$\frac{d}{dt}\left[M(k^2+l^2)\dot{\theta}\right] + Mgl\sin\theta = 0.$$

or, $M(k^2+l^2)\ddot{\theta} + Mgl\sin\theta = 0.$

or, $(k^2+l^2)\ddot{\theta} + gl\sin\theta = 0.$

or, $\boxed{\ddot{\theta} + \frac{g\,l\sin\theta}{(k^2+l^2)} = 0.}$ —— (3)

This is the equation of motion of the Compound pendulum.

If θ is small, then $\sin\theta \simeq \theta$, then,

$$\ddot{\theta} + \frac{gl\theta}{(k^2+l^2)} = 0.$$

& its Time period is $T = 2\pi\sqrt{\frac{k^2+l^2}{lg}}$

$$\boxed{T = 2\pi\sqrt{\frac{\left(\frac{k^2}{l}+l\right)}{g}}} \quad \text{—— (4)}$$

=x=x=